1. The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.

A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs).

Table

Description automatically generated

Let the continuous decision variables *x*1,*x*2,*x*3, and *x*4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

|  |  |
| --- | --- |
| 1. | No more than two of the products can be produced. |
| 2. | Either product 3 or 4 can be produced only if either product 1 or 2 is produced. |
| 3. | Either 5*x*1 + 3*x*2 + 6*x*3 + 4*x*4 ≤ 6,000  or 4*x*1 + 6*x*2 + 3*x*3 + 5*x*4 ≤ 6,000. |

1. Develop a Mixed Integer Programming (MIP) model.
2. Use Excel solver to solve the problem.

2. The MSU Company has developed two new products for possible inclusion in its product line for the upcoming Black Friday. Setting up the production facilities to begin production would cost $50,000 for product 1 and $80,000 for product 2. Once these costs are covered, the products would generate a unit profit of $10 for product 1 and $15 for product 2.

The MSU Company has two factories that have the capability to manufacture the products. However, to avoid incurring duplicate start-up costs, only one factory will be utilized, and the selection will be based on maximizing profits. If both new products are produced, for organizational purposes, the same factory will be used for both.

Factory 1 has the capacity to manufacture Product 1 at a rate of 50 units per hour, and Product 2 at a rate of 40 units per hour. Factory 2 has a lower capacity and can produce Product 1 at a rate of 40 units per hour and Product 2 at a rate of 25 units per hour. There are 500 hours of production time available in Factory 1 and 700 hours of production time available in Factory 2 before Black Friday that can be used to manufacture these products.

It is not known whether these two products would be continued after Black Friday. Therefore, the problem is to determine how many units (if any) of each new product should be produced before Black Friday to maximize the total profit.

|  |  |
| --- | --- |
| (a) | Formulate an MIP model for this problem. |
| (b) | Use Python to solve this model. |

3. Cessna manufactures small jet planes that are sold to businesses for the purpose of transporting their top-level executives. In order to meet the specific requirements of these executives, some customers request a personalized design for the planes they purchase. This results in a significant initial expense to begin the production of these customized airplanes.

Cessna has received purchase orders from three clients with tight deadlines, but due to the high demand for their products, the company's manufacturing facilities are currently occupied with fulfilling previous orders. As a result, Cessna needs to make a decision on how many airplanes, if any, they can produce for each of the three clients.

The following table provides the necessary information for decision-making. The first row indicates the initial cost required to begin producing airplanes for each of the three customers. The second row displays the net revenue earned from each airplane produced, which is the purchase price minus the marginal production cost. The third row illustrates the percentage of production capacity utilized for each airplane to be produced. The last row specifies the maximum number of airplanes requested by each customer, but a lower number may be accepted.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Customer** | | |
|  | **1** | **2** | **3** |
| Start-up cost | $3 million | $2 million | 0 |
| Marginal net revenue | $2 million | $3 million | $0.8 million |
| Capacity used per plane | 20% | 40% | 20% |
| Maximum order | 3 planes | 2 planes | 5 planes |

Cessna aims to decide on the number of airplanes, if any, that they should manufacture for each customer in order to maximize the company's overall profit. This profit will be calculated by subtracting the start-up costs from the total net revenue earned.

|  |  |
| --- | --- |
| (a) | Formulate a model with both integer variables and binary variables for this problem. |
| (b) | Use Excel Solver to solve this model. |

4. Reconsider the third question. Upon a more thorough examination of the costs and revenues associated with producing airplanes for each customer in the third problem, it has been determined that the potential profits cannot be accurately represented using just a start-up cost and a constant net revenue per airplane produced. Instead, the profits associated with each customer are presented in the table below:

|  |  |  |  |
| --- | --- | --- | --- |
| Airlines Produced | Profit from Customer | | |
| Customer 1 | Customer 2 | Customer 3 |
| 0 | 0 | 0 | 0 |
| 1 | -$1 million | $1 million | $1 million |
| 2 | $2 million | $5 million | $3 million |
| 3 | $4 million |  | $5 million |
| 4 |  |  | $6 million |
| 5 |  |  | $7 million |

|  |  |
| --- | --- |
| (a) | Formulate a BIP model for this problem. |
| (b) | Use Excel solver to solve the model formulated in part (*a*). What is the optimal number of airplanes produced for each customer? |

**Note:** Question 4 is a new problem. The constraints in question 3 will not be included in the mathematical model in question 4.

5. Samaritan Care Services must schedule nurses so that the hospitals’ patients are provided with adequate care while at the same time keeping down the costs. The minimum number of nurses to have on hand for various times of the day is provided in the table below:

|  |  |  |
| --- | --- | --- |
| Shift | Time | Minimum Number of Nurses Needed |
| 1 | 12:00 A.M. - 4:00 A.M. | 5 |
| 2 | 4:00 A.M. - 8:00 A.M. | 12 |
| 3 | 8:00 A.M. - 12:00 P.M. | 14 |
| 4 | 12:00 P.M. - 4:00 P.M. | 8 |
| 5 | 4:00 P.M. - 8:00 P.M. | 14 |
| 6 | 8:00 P.M. - 12:00 A.M. | 10 |

The nurse scheduling problem seeks to find the minimum total number of nurses required to provide adequate care. Nurses start work at the beginning of one of the 4-hour shifts and work for 8 hours.

Formulate and solve the nurse scheduling problem as an integer program for one day **using Python**. (You must clearly define decision variables and generate the objective function and constraints).